

MIDTERM REVIEW---HONORS

NAME: key

REAL NUMBERS --- Section 1.1

- 1) Given the set: $\left\{-3, \sqrt{12}, \frac{16}{4}, -\sqrt{64}, 7.121121112..., 7\frac{2}{5}, -9.55, 0, 8, \frac{13}{72}, 4.\overline{1237}, .72\overline{3}\right\}$

List ALL the elements of the set that belong to each set below.

- a) Natural Numbers (N) $\frac{16}{4}, 8$
b) Whole Numbers (W) $\frac{16}{4}, 0, 8$
c) Integers (I) $-3, \frac{16}{4}, -\sqrt{64}, 0, 8$
d) Rational Numbers (Q) $-3, \frac{16}{4}, \sqrt{4}, 7\frac{2}{5}, -9.55, 0, 8, \frac{13}{72}, 4.\overline{1237}, .72\overline{3}$
e) Irrational Numbers (Ir) $\sqrt{12}, 7.121121112...$
f) Real Numbers (R) $-3, \sqrt{12}, \frac{16}{4}, -\sqrt{64}, 7.121121112..., 7\frac{2}{5}, -9.55, 0, 8, \frac{13}{72}, 4.\overline{1237}, .72\overline{3}$

- 2) Match each equation to the name of the property illustrated by the equation.

- 2 a) $9 + (\sqrt{5} + 0) = 9 + (0 + \sqrt{5})$ 1) Associative Property of Multiplication
1 b) $2 + 7\left(\frac{2}{3} \cdot \frac{3}{2}\right) = 2 + \left(7 \cdot \frac{2}{3}\right) \cdot \frac{3}{2}$ 2) Commutative Property of Addition
5 c) $2 + 7 \cdot 1 = 2 + 7$ 3) Inverse Property of Multiplication
4 d) $6(-4 + 4) = 6(0)$ 4) Inverse Property of Addition
7 e) $6(-4 + 4) = -24 + 24$ 5) Identity Property of Multiplication
6 f) $9 + (\sqrt{5} + 0) = 9 + \sqrt{5}$ 6) Identity Property of Addition
3 g) $2 + 7\left(\frac{2}{3} \cdot \frac{3}{2}\right) = 2 + 7 \cdot 1$ 7) Distributive Property

- 3) Complete the following using the order of operations. Do only **ONE STEP** at a time.

$$\begin{aligned} & 8 + (-3^2 + 3) \div 2 \cdot 4 - 6 \\ & 8 + (-9 + 3) \div 2 \cdot 4 - 6 \\ & 8 - 6 \div 2 \cdot 4 - 6 \\ & 8 - 3 \cdot 4 - 6 \\ & 8 - 12 - 6 \\ & \quad -4 - 6 = \boxed{-10} \end{aligned}$$

ORDER AND ABSOLUTE VALUE --- Section 1.2

- 4) Evaluate the following expression given $x = -3$ and $y = -5$.

$$\frac{2|y| - x^3}{|y^2 + 3x|} = \frac{2|-5| - (-3)^3}{|(-5)^2 + 3(-3)|} = \frac{2 \cdot 5 - (-27)}{|25 - 9|} = \frac{10 + 27}{16} = \boxed{\frac{37}{16}}$$

5) Given $x > 3$, write each of the following without absolute value bars.

- a. $|x^2 - 9| = \underline{x^2 - 9}$
 b. $|9 - x^2| = \underline{-(9 - x^2) = x^2 - 9}$
 c. $|3 - x| = \underline{-(3 - x) = x - 3}$
 d. $|-x^2 - 2| = \underline{-(-x^2 - 2) = x^2 + 2}$

RATIONAL EXPONENTS --- Sections 1.3/1.6

6) Evaluate each of the following. Show the problem you use to complete each.

- a. $(-5)^2 = \underline{(-5)(-5) = 25}$
 b. $-5^2 = \underline{-(5)(5) = -25}$

7) Simplify (no like bases, no negative exponents) each of the following.

a. $\frac{(2k)^{-3}(k^{-5})^{-1}}{(6k^{-2})^{-1}(k^3)^{-6}} = \frac{k^6}{(2k)^3(6k^2)(k^{-18})}$
 $= \frac{6k^2}{8k^{-18}} = \boxed{\frac{3k^{20}}{4}}$

b. $\frac{3x^3y^{-\frac{1}{5}}z^{\frac{2}{9}}}{4x^{-2}z^{\frac{1}{3}}} = \boxed{\frac{3x^5}{4y^{\frac{1}{5}}z^{\frac{1}{9}}}}$

$$\frac{2}{9} - \frac{1}{3} = -\frac{1}{9}$$

$$\frac{2}{9} - \frac{2}{9} = -\frac{1}{9}$$

8) Simplify $243^{-\frac{2}{5}}$. $\left(\frac{1}{243}\right)^{\frac{2}{5}} = \left(\frac{1}{\sqrt[5]{3^5}}\right)^2 = \left(\frac{1}{3}\right)^2 = \boxed{\frac{1}{9}}$

RADICAL EXPRESSIONS --- Section 1.7

9) Simplify: $\sqrt[4]{x^{12}y^{46}z^{103}} = \boxed{x^3y^{11}z^{25}\sqrt[4]{y^2z^3}}$

10) Add /Subtract: $-3\sqrt{54} - 2\sqrt{18} + 5\sqrt{96}$

$$-3\sqrt{9 \cdot 6} - 2\sqrt{9 \cdot 2} + 5\sqrt{16 \cdot 6}$$

$$-9\sqrt{6} - 6\sqrt{2} + 20\sqrt{6}$$

$$\boxed{11\sqrt{6} - 6\sqrt{2}}$$

11) Rationalize the denominator:

$$a) \sqrt[3]{\frac{5}{24}} = \frac{\sqrt[3]{5}}{\sqrt[3]{24}} = \frac{\sqrt[3]{5}}{2\sqrt[3]{3}} = \frac{\sqrt[3]{45}}{6}$$

$$b) \frac{2+2\sqrt{6}}{4-\sqrt{6}} \cdot \frac{4+\sqrt{6}}{4+\sqrt{6}} = \frac{8+2\sqrt{6}+8\sqrt{6}+12}{16-6} = \frac{20+10\sqrt{6}}{10} = \boxed{2+\sqrt{6}}$$

COMPLEX NUMBERS --- Section 2.3

$$12) \text{ Multiply: } 3\sqrt{-27} \cdot 2\sqrt{-60} = 3\sqrt{-1 \cdot 3^3} \cdot 2\sqrt{-1 \cdot 4 \cdot 5} = 9i\sqrt{3} \cdot 4i\sqrt{5} = 36i^2\sqrt{45} = \boxed{-108\sqrt{5}}$$

$$13) \text{ Simplify: } i^{2306} = i^2 = \boxed{-1}$$

$$14) \text{ Add/Subtract: } (7-3i) + (-4+7i) + (\bar{6}+8i) = \boxed{-3+12i}$$

$$15) \text{ Rationalize: } \frac{4+3i}{2-5i} \cdot \frac{2+5i}{2+5i} = \frac{8+20i+6i-15}{4+25} = \frac{-7+26i}{29} = \boxed{-\frac{7}{29} + \frac{26i}{29}}$$

POLYNOMIALS --- Section 1.3

*16) Give the degree and the most specific classification for: $3x^3 + 5x^2y^4 - 8xyz$

Degree 6
trinomial
multi-variate

$$17) \text{ Add/Subtract: } (3x^2 - 2x + 3) + 4(5x^2 - 2x + 4) + (-2x^2 + 6x + 7) \\ (3x^2 - 2x + 3) + (20x^2 - 8x + 16) + (-2x^2 - 6x + 7) \\ \boxed{21x^2 - 16x + 26}$$

$$18) \text{ Multiply: } (3x+5)(2x^2-3x+5) \\ \begin{array}{r} 6x^3 - 9x^2 + 15x \\ 10x^2 - 15x + 25 \\ \hline \boxed{6x^3 + x^2 + 25} \end{array}$$

- 19) Divide: $(4x^4 - 4x^3 - 1) \div (2x^2 - 1)$

$$\begin{array}{r} 2x^2 - 2x + 1 \\ 2x^2 - 1 \overline{) 4x^4 - 4x^3 - 1} \\ \underline{-4x^4} -1 \\ +2x^2 -1 \\ \underline{-2x^2} +1 \\ -2x +1 \\ \underline{-2x^2} +1 \\ +1 \end{array}$$

$$\boxed{2x^2 - 2x + 1 - \frac{2x}{2x^2 - 1}}$$

- 20) Square the trinomial: $(3x^3 - 7z^8 + 2)^2$

$$\begin{array}{r} (3x^3 - 7z^8 + 2)(3x^3 - 7z^8 + 2) = \\ 9x^6 - 21x^3z^8 + 6x^3 \\ -21x^3z^8 + 49z^{16} - 14z^8 \\ +6x^3 - 14z^8 + 4 \end{array}$$

$$\boxed{9x^6 - 42x^3z^8 + 12x^3 + 49z^{16} - 14z^8 + 4}$$

THE BINOMIAL THEOREM --- Sections 1.3 and 8.4

- 21) Use **Pascal's triangle** to expand $(4x + 3)^5$.

$$\begin{array}{cccccc} & & 1 & & & \\ & 1 & & 1 & & \\ & & 1 & & 2 & & 1 \\ 1 & & 3 & & 3 & & 1 \\ & 1 & & 4 & & 6 & & 4 & & 1 \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & 5 \end{array}$$

$$\begin{array}{l} (4x)^5(3)^0 + 5(4x)^4(3)^1 + 10(4x)^3(3)^2 + 10(4x)^2(3)^3 + 5(4x)(3)^4 + (4x)^0(3)^5 \\ \boxed{1024x^5 + 3840x^4 + 5760x^3 + 4320x^2 + 1620x + 243} \end{array}$$

COUNTING THEORY --- Section 8.6

- 22) Write the factorial notation and evaluate the following:

a) $P(10, 7)$

b) $C(10, 7)$

$${}_{10}P_7 = \frac{10!}{(10-7)!} = \frac{10!}{3!} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = \boxed{604800}$$

$$C(10, 7) = \frac{10!}{7!(10-7)!} = \frac{10!}{7!3!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = \boxed{120}$$

- 23) A teacher has 5 books

- a) How many ways can she arrange 3 of these on her desk?

order matters ${}_{5}P_3 = \frac{5!}{(5-3)!} = \boxed{60}$

- b) How many ways can she pick 3 of these give to another teacher?

order does not matter ${}_{5}C_3 = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = \boxed{10}$

- 24) A math class consisting of 18 males and 12 females is preparing for the midterm exam.

- a) How many study groups of 5 can be formed?

$${}_{30}C_5 = \boxed{142506} \text{ groups}$$

- b) How many study groups of 2 males and 3 females can be formed?

$${}_{18}C_2 \text{ AND } {}_{12}C_3 = 153 \cdot 220 = \boxed{33660} \text{ groups}$$

FACTORING POLYNOMIALS --- Section 1.4

25) Factor each of the following polynomials COMPLETELY:

$$a) 27x^{27} - 64y^{15} = (3x^9 - 4y^5)(9x^{18} + 12x^9y^5 + 16y^{10})$$

$$b) 20x^2 - 28x - 3 = (2x - 3)(10x + 1)$$

$$c) 25x^2 - 60x + 36 - 81y^2 = (5x - 6)^2 - 81y^2 = [(5x - 6) - 9y][(5x - 6) + 9y] = (5x - 9y - 6)(5x + 9y - 6)$$

$$d) 9x^2(2x^4 + 7)^3(x + 3)^7 - 2(2x^4 + 7)^4(5)(x + 3)^6 = \frac{(2x^4 + 7)^3(x + 3)^6(9x^3 + 27x^2 - 20x^4 - 70)}{(2x^4 + 7)^3(x + 3)^6(-20x^4 + 9x^3 + 27x^2 - 70)} = \frac{9x^3 + 27x^2 - 20x^4 - 70}{-(2x^4 + 7)(x + 3)^6(20x^4 - 9x^3 - 27x^2 + 70)}$$

RATIONAL EXPRESSIONS --- Section 1.5

$$26) \text{ Reduce: } \frac{81 - 16x^2}{4x^2 - x - 18} = \frac{(9 - 4x)(9 + 4x)}{(4x - 9)(x + 2)} = \boxed{\frac{-(9 + 4x)}{x + 2}}$$

$$27) \text{ Divide: } \frac{x^2 + 4x + 4}{3x + 12} \div \frac{x + 2}{x^2 + 4x} = \frac{(x + 2)(x + 2)}{3(x + 4)} \cdot \frac{x(x + 4)}{(x + 2)} = \boxed{\frac{x(x + 2)}{3}}$$

$$28) \text{ Subtract: } \frac{x - 7}{x^2 + 4x - 5} - \frac{x - 9}{x^2 + 3x - 10} =$$

$$\frac{(x - 7)(x - 2)}{(x + 5)(x - 1)(x - 2)} - \frac{(x - 9)(x - 1)}{(x + 5)(x - 2)(x - 1)} =$$

$$\frac{x^2 - 9x + 14}{(x + 5)(x - 1)(x - 2)} - \frac{x^2 - 10x + 9}{(x + 5)(x - 1)(x - 2)} = \frac{x + 5}{(x + 5)(x - 1)(x - 2)} = \boxed{\frac{1}{(x - 1)(x - 2)}}$$

$$29) \text{ Simplify: } \frac{\frac{1}{x-1} - \frac{6}{x}}{\frac{1}{x}} = \frac{\frac{x}{x(x-1)} + \frac{-6(x-1)}{x(x-1)}}{\frac{1}{x}} = \frac{\frac{x}{x(x-1)} + \frac{-6x+6}{x(x-1)}}{\frac{1}{x}} = \frac{\frac{-5x+6}{x(x-1)}}{\frac{1}{x}}$$

$$= \frac{-5x+6}{x(x-1)} \cdot \frac{x}{1} = \boxed{\frac{-5x+6}{x-1}}$$

EQUATIONS/APPLICATIONS --- Sections 2.1, 2.2, 2.4, 2.5, 2.6

30) Solve the equation: $\frac{y+3}{y-6} + \frac{y-2}{y+2} = \frac{2y^2+9}{y^2-4y-12} = \left[\frac{y+3}{y-6} + \frac{y-2}{y+2} = \frac{2y^2+9}{(y-6)(y+2)} \right] (y-6)(y+2)$

$$(y+3)(y+2) + (y-2)(y-6) = 2y^2+9$$

$$y^2+5y+6 + y^2-8y+12 = 2y^2+9$$

$$-3y+18=9$$

$$-3y=-9$$

$$y=3$$

31) Solve the equation $\frac{2b}{c-1} = b-d$; for c

$$\frac{2b}{b-d} = c-1$$

$$\frac{2b}{b-d} + \frac{b-d}{b-d} = c$$

$$\frac{2b}{b-d} + 1 = c$$

$$\frac{3b-d}{b-d} = c \quad b \neq d$$

32) Solve the equation: $5(x^2-1) = 2x(x+6)$

a) Using the quadratic formula

$$5x^2-5 = 2x^2+12x$$

$$3x^2-12x-5=0$$

$$a=3 \quad b=-12 \quad c=-5$$

$$x = \frac{12 \pm \sqrt{144-4(3)(-5)}}{6}$$

$$x = \frac{6 \pm 2\sqrt{51}}{3}$$

$$x = \frac{12 \pm \sqrt{144+60}}{6} = \frac{12 \pm \sqrt{204}}{6} = \frac{12 \pm 2\sqrt{51}}{6}$$

b) By completing the square

$$3x^2-12x-5=0$$

$$3x^2-12x=5$$

$$x^2-4x=\frac{5}{3}$$

$$x^2-4x+4=\frac{5}{3}+\frac{12}{3}$$

$$(x-2)^2=\frac{17}{3} \rightarrow x-2=\pm\sqrt{\frac{17}{3}}$$

$$x-2=\pm\sqrt{\frac{17}{3}}$$

$$x = \frac{6 \pm \sqrt{51}}{3}$$

33) Calculate the discriminate and tell what kind and how many solutions each of the following quadratic equations has.

a) $4x^2-4x=3$ $4x^2-4x-3=0$ $a=4 \quad b=-4 \quad c=-3$

$$b^2-4ac = 16-4(4)(-3) = 16+48 = 64$$

2 Real Rational

b) $4x^2-12x=-9$ $4x^2-12x+9=0$ $a=4 \quad b=-12 \quad c=9$

$$b^2-4ac = 144-4(4)(9) = 144-144 = 0$$

1 Real Rational

c) $4x^2-4x=10$ $4x^2-4x-10=0$ $a=4 \quad b=-4 \quad c=-10$

$$b^2-4ac = 16-4(4)(-10) = 16+160 = 176$$

2 Real Irrational

d) $4x^2-12x=-10$ $4x^2-12x+10=0$ $a=4 \quad b=-12 \quad c=10$

$$b^2-4ac = 144-4(4)(10) = 144-160 = -16$$

2 Non-Real Imaginary

34) Solve the equation: $\sqrt{3x+4}-1=\sqrt{x+5}$

$$3x+4-2\sqrt{3x+4}+1=x+5$$

$$-2\sqrt{3x+4}=-2x$$

$$\sqrt{3x+4}=x$$

$$3x+4=x^2$$

$$0=x^2-3x-4$$

$$0=(x-4)(x+1)$$

$$x=4 \quad x=-1$$

check $\frac{\sqrt{16}-1}{4-1}=\frac{3}{3}=1$

$\frac{\sqrt{1}-1}{-1-1}=\frac{0}{-2}=0 \neq 2$

$$\{4\}$$

35) Solve the equation: $2x^{-4} - 5x^{-2} = 3$

let $y = x^{-2}$

$$2y^2 - 5y - 3 = 0$$

$$(2y + 1)(y - 3) = 0$$

$$y = -\frac{1}{2}$$

$$y = 3$$

$$x^{-2} = -\frac{1}{2}$$

$$x^{-2} = 3$$

$$x^2 = -2$$

$$x^2 = \frac{1}{3}$$

$$x = \pm i\sqrt{2}$$

$$x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$$\left\{ \pm i\sqrt{2}, \pm \frac{\sqrt{3}}{3} \right\}$$

36) If Andrew can paint the house in 8 hours and Pat can paint the house in 6 hours, how long will it take them to paint the house if they work together?

	Part/hr	time (hr)	Part completed
Andrew	$\frac{1}{8}$	x	$\frac{x}{8}$
Pat	$\frac{1}{6}$	x	$\frac{x}{6}$

let x = time spent working together (hrs)

Andrew's Part + Pat's Part = whole

$$\left(\frac{x}{8} + \frac{x}{6} = 1 \right) 48$$

$$6x + 8x = 48$$

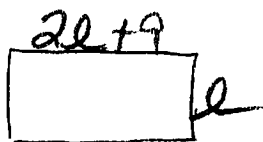
$$14x = 48$$

$$x = 3\frac{4}{7}$$

$$\frac{4}{7} \cdot 60 = 34.28$$

Together it would take them 3 hrs 34 min

37) The width of the rectangle is 9 inches longer than twice the length. If the area of the rectangle is 18 square inches, what are the dimensions of the rectangle?



let l = length of rectangle

let width = $2l + 9$

$$A = (\text{length})(\text{width})$$

$$18 = (l)(2l+9)$$

$$18 = 2l^2 + 9l$$

$$0 = 2l^2 + 9l - 18$$

$$0 = (2l - 3)(l + 6)$$

$$l = \frac{3}{2} \quad l = -6$$

The length is 1.5 inches
the width is 12 inches

- 38) Two friends decide to meet in Chicago to attend a Cub's baseball game. Rob travels 310 miles in the same time that Carl travels 295 miles. Rob's trip uses more interstate highways and he can average 3 mph more than Carl. What is Rob's average speed?

	Rate (mph)	Time (hr)	Distance (miles)
Rob	x	$\frac{310}{x}$	310
Carl	$x-3$	$\frac{295}{x-3}$	295

let x = rate at which Rob travels (mph)

$t = \frac{D}{r}$

$$\text{Time Rob Travels} = \text{Time Carl travels}$$

$$\frac{310}{x} = \frac{295}{x-3}$$

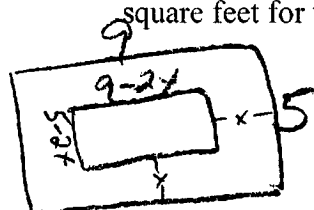
$$310x - 930 = 295x$$

$$15x = 930$$

$$x = 62$$

Rob's avg speed is 62 mph

- 39) A landscape architect has included a rectangular flower bed measuring 9 feet by 5 feet in her plans for a new building. She wants to use two colors of flowers in the bed, one in the center and the other for a border of the same width on all four sides. If she has enough plants to cover 24 square feet for the border, how wide can the border be?



let x = width of border (ft)

Area of border can be 24 ft^2

$$\text{Area of border} = \text{Area of entire garden} - \text{Area of center garden}$$

$$24 = (9)(5) - (9-2x)(5-2x)$$

$$24 = 45 - [45 - 28x + 4x^2]$$

$$24 = 45 - 45 + 28x - 4x^2$$

$$4x^2 - 28x + 24 = 0$$

$$x^2 - 7x + 6 = 0$$

$$(x-6)(x-1) = 0$$

not feasible $\rightarrow x \neq 6 \quad x = 1$

The border should be 1 ft

- 40) Including an 8% sales tax, an inn charges \$162 per night. Find the inn's nightly cost before the tax is added.

let x = cost of stay at 19 before tax (pretax)

$$\text{Final charge} = \text{pre-tax} + \text{tax}$$

$$162 = x + .08x$$

$$162 = 1.08x$$

$$150 = x$$

The pretax charge is \$150